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Calculation of two-dimensional and axisymmetric bluff-body potential flow

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A numerical method incorporating some of the ideas underlying the wake source model of Parkinson & Jandali (1970) is presented for calculating the incompressible potential flow external to a bluff body and its wake. The effect of the wake is modelled by placing sources on the rear of the wetted surface of the body. Unlike Parkinson & Jandali's method, however, the body shapes that can be treated are not limited by the restrictions imposed by the use of conformal transformation. In the present method the wetted surface of the body is represented by a distribution of discrete vortices. Good agreement has been found between the pressure distributions predicted by the numerical method and the analytic expressions of Parkinson & Jandali for a 'two-dimensional' circular cylinder and flat plate. A flat plate at incidence and other asymmetric two-dimensional flows have also been treated. The method has been extended to axisymmetric bluff bodies and the results show good agreement with measured pressure distributions on a circular disk and a sphere.

1. Introduction

No theory is capable of predicting all the aspects of the flow past bluff bodies. Considerable progress has been made, however, in the calculation of the time-averaged potential flow about simple two-dimensional bluff shapes. In these solutions the thin separating shear layers are replaced by free streamlines and the potential flow external to these streamlines and the wetted surface of the body is found. The pressure on the part of the body in the separated region, the base pressure, has to be found from experiment. Unless the bluff body has sharp edges where separation is expected to occur, the separation points also need to be found from experiment. The positions of the free streamlines are initially unknown but this is overcome by specifying a velocity distribution along them so that their positions are fixed in the complex velocity and complex potential planes. The solution is found by conformally transforming from these planes to the physical plane. The free-streamline method of Roshko (1954) and the more general methods of Woods (1955) and Wu (1962), all using the above approach, show good agreement between measured and predicted time-averaged surface measurements on simple two-dimensional bluff bodies.

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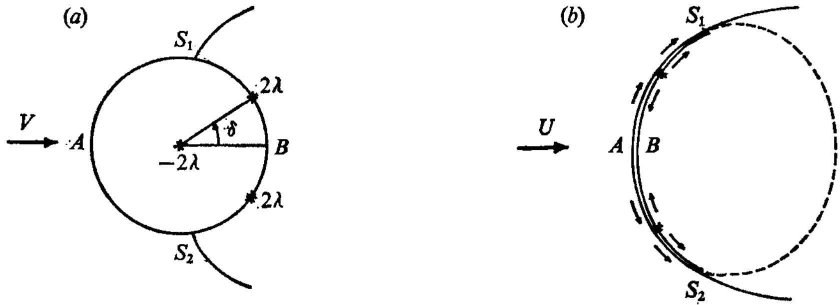

FIGURE 1. (*a*) Parkinson & Jandali transform (ζ) plane and (*b*) physical (z) plane.

More recently Parkinson & Jandali (1970, referred to as P & J) have suggested an alternative approach for specifying the free-streamline positions using two surface sources. They consider a uniform flow past a circular contour in some transform plane ζ (see figure 1). Added to this flow are two sources located symmetrically on the contour with images at the centre. The flow from the surface sources creates stagnation points on either side of the circular boundary. A transformation is used which maps this contour onto a slit in the physical plane corresponding to the wetted surface of the body. The transform is chosen so as to make the two stagnation points critical points in order that the flow in the real plane should be tangential to the upstream surface as it leaves the body. The sources do not model the mean flow in the wake and flow between the separating streamlines is ignored. The purpose of the sources is to model the effect of the wake on the potential flow and to set the separation velocities. The P & J method gives just as good agreement with experiment as other free-streamline theories but has the advantage of being simpler to apply. This method is limited, however, to two-dimensional shapes and depends on the knowledge of suitable transformations. The aim of the work described here was to develop a method for calculating potential flow about bluff bodies of arbitrary shape which retained the modelling of the effect of the wake by surface sources. Inspection of the P & J approach shows that in the real plane the wetted surface of the body consists of a sheet of vorticity with two superimposed sources. The vorticity falls to zero at the separation points, since the flow leaves tangentially, and has infinities at the source positions in order to satisfy the condition of zero normal velocity on the front face of the body. Abandoning conformal transformation, we see that the problem reduces to finding the distribution of vorticity over the wetted surface, together with the source positions and strengths, consistent with the boundary conditions of zero normal velocity on the wetted surface and given separation positions and base pressure. The determination of vorticity distributions on thin aerofoils has been successfully accomplished by the use of vortex-lattice methods and it is proposed to use a similar approach to the present problem. It will be possible to check the accuracy of the numerical method, for simple two-dimensional shapes, by comparison with the P & J analytic method.

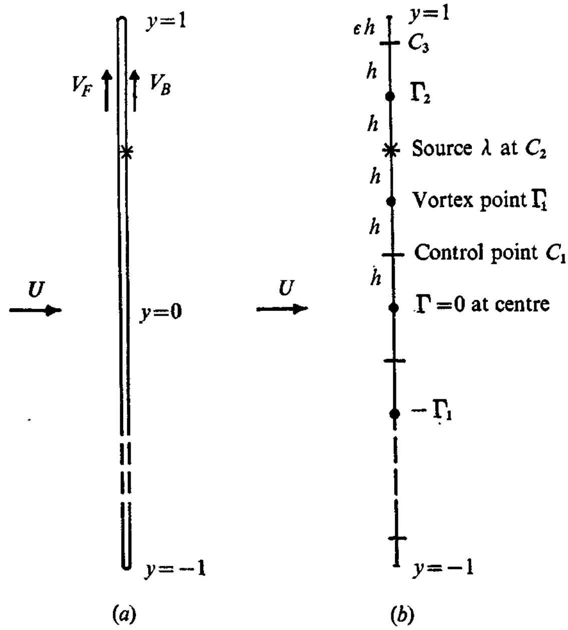

FIGURE 2. The normal flat plate. (*a*) P & J model. (*b*) Vortex-lattice model, $N = 5$.

2. Vortex-lattice discretization for a 'two-dimensional' flat plate normal to the free stream

Initially we shall consider the flow about a 'two-dimensional' flat plate normal to the free stream. The vorticity γ on the plate is given by

$$\gamma(y) = \tfrac{1}{2}(V_B - V_F),$$

where V_B and V_F are the velocities on the back and front of the plate respectively (figure 2*a*). The P & J solution for the velocity field around a normal flat plate leads to the vorticity distribution

$$\gamma(y) = -y(1-y^2)^{\frac{1}{2}}/(y^2 - y_s^2), \tag{2.1}$$

where $y = \pm 1$ are the edges of the plate and $y = \pm y_s$ are the source positions. We wish to obtain an approximation to this distribution by using a numerical discretization method.

The vortex-lattice approach consists of replacing a continuous distribution of vorticity by a set of discrete vortices. For the normal flat plate we shall make use of symmetry to halve the number of unknowns, although this is not done in the general method derived from this analysis. Thus we place N vortices of strengths Γ_j at $y = 2hj$, $j = 1, \ldots, N$, where h is a constant (figure 2*b*). The normal velocity at the wetted surface of the plate is then sampled and set equal to zero at $N+1$ control points, placed at $y = (2i-1)h, i = 1, \ldots, N+1$. A source of strength λ is placed at one of the control points, $y = (2i_s - 1)h$, say. When the normal velocity is evaluated at this point, the source is treated as if it were uniformly distributed over the surface between the two adjacent vortices with density $\lambda/2h$. Its normal-velocity contribution is then $\pi\lambda/2h$. At all other control points the source is treated as a point singularity and thus, on the flat plate, provides no contribution to the normal velocity. Then sampling the normal velocity at $N+1$ control points, and making the free-stream velocity unity, gives

$$\left.\begin{aligned} &\sum_{j=1}^{N}\left[\frac{j}{[(i-\frac{1}{2})^2-j^2]}\right]\frac{\Gamma_j}{h} = 1 \quad \text{for} \quad i = 1,\ldots,N+1, \quad i \neq i_s \\ \text{and} \quad &\sum_{j=1}^{N}\left[\frac{j}{[(i_s-\frac{1}{2})^2-j^2]}\right]\frac{\Gamma_j}{h}+\frac{\pi\lambda}{2h} = 1 \quad \text{for} \quad i = i_s. \end{aligned}\right\} \tag{2.2}$$

(The 2π normally associated with the velocities has been incorporated in Γ_j and λ.) The first N equations will determine the N unknowns Γ_j and the last equation provides the value of λ. The half-width of the plate in terms of h is

$$(2N+1+\epsilon)\,h = 1, \tag{2.3}$$

where ϵh is the distance between the last control point and the separation point. We shall use an analysis based on that employed by Doe (1971) for thin aerofoils to show that the vortex-lattice approximation rapidly converges to the correct solution provided that $\epsilon = \frac{1}{2}$.

Because the matrix of coefficients of the first N equations can be written in the form of the Hilbert segment, for which an inverse is known, the solution of these equations can be shown to be

$$\begin{aligned} \frac{\Gamma_j}{h} = \frac{4}{\pi^2}\,&\frac{j(N+\frac{1}{2}-j)^{(\frac{1}{2})}(N+\frac{1}{2}+j)^{(\frac{1}{2})}}{(i_s-\frac{1}{2}-j)(i_s-\frac{1}{2}+j)} \\ &\times\left[\sum_{i=1}^{N+1}\left(\frac{(i_s-i)(i_s+i-1)(N+\frac{1}{2}-i)^{(-\frac{1}{2})}(N-\frac{1}{2}+i)^{(-\frac{1}{2})}}{(i-\frac{1}{2}-j)(i-\frac{1}{2}+j)}\right)\right], \end{aligned} \tag{2.4}$$

where the notation $x^{(a)} = x!/(x-a)!$ is used and the factorial symbol is defined for non-integer values of x through the Gamma function: $x! = \Gamma(x+1)$.

The factorial function $x^{(a)}$ has the property

$$[x+\tfrac{1}{2}^{(a-1)}]^{(a)} \sim x^a \quad \text{as} \quad x \to \infty. \tag{2.5}$$

So, for N large, (2.4) becomes, away from the edges of the plate,

$$\frac{\Gamma_j}{h} = \frac{4}{\pi^2}\frac{j[(N+\frac{3}{4})^2-j^2]^{\frac{1}{2}}}{[(i_s-\frac{1}{2})^2-j^2]}\sum_{i=1}^{N+1}\left(\frac{(i_s-\frac{1}{2})^2-(i-\frac{1}{2})^2}{[(i-\frac{1}{2})^2-j^2]\,[(N+\frac{3}{4})^2-(i-\frac{1}{2})^2]^{\frac{1}{2}}}\right). \tag{2.6}$$

If we write $y = 2jh, y' = (2i-1)\,h$ and $dy' = 2h$ and let

$$(N+\tfrac{3}{4})\,2h = 1 \tag{2.7}$$

(2.6) becomes in the limit

$$\pi\Gamma_j/2h = -y(1-y^2)^{\frac{1}{2}}/(y_s^2-y^2). \tag{2.8}$$

The right-hand side is the same as the result (2.1) of the continuous P & J model. Comparison of (2.3) and (2.7) shows that this result requires that $\epsilon = \frac{1}{2}$.

The limiting form (2.6) is approached very rapidly. For example, the percentage error in using $(x-\frac{1}{4})^{(\frac{1}{2})}$ instead of $x^{\frac{1}{2}}$ at $x = \frac{3}{4}$, which corresponds to the vortex

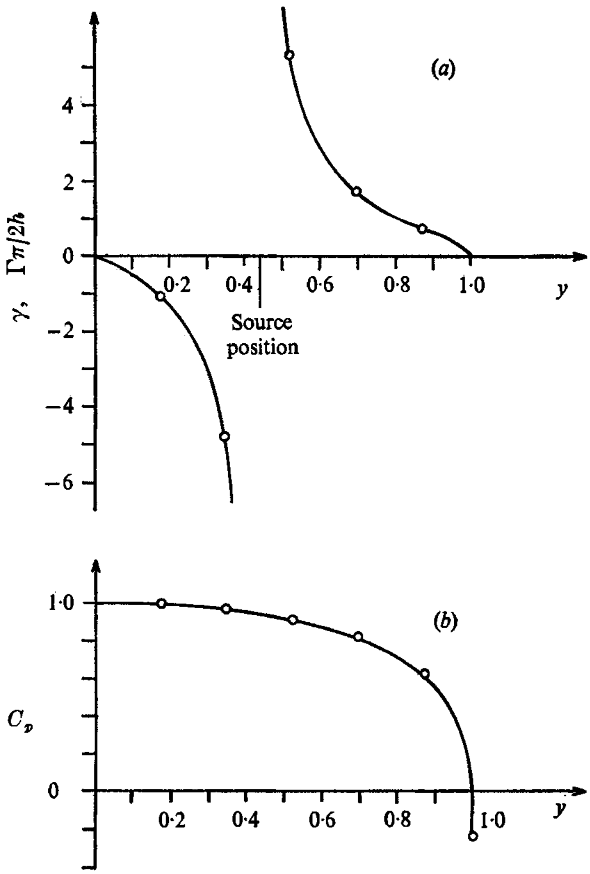

FIGURE 3. Results for flat plate with $N = 5$ and source at C_3. (*a*) Vorticity distribution. (*b*) Pressure distribution. ——, theory; ○, numerical.

position closest to the edge of the plate, is 2 % and this reduces to only 0·5 % at the next vortex position. Similarly the difference between $x^{-\frac{1}{2}}$ and $(x-\frac{3}{4})^{(-\frac{1}{2})}$ at $x = \frac{1}{4}$, the control point nearest to the edge, is 11·4 % but at the next control point it is only 0·9 %. Thus the restrictions imposed by taking the above limits are not severe and the discrete model should give an accurate representation over the whole plate, even close to the edges. Nor is it necessary to have N very large, as illustrated by figure 3(*a*), which gives results for $N = 5$ compared with the P & J analytic solution.

Once the equations have been solved for the vortex and source strengths, the surface pressure is evaluated at the vortex positions. The tangential velocity at a vortex point is obtained by summing the contributions from the free stream, the vortices and from the sources. The contribution from the vortex situated at the point itself is treated in a similar way to that from a source, described earlier, i.e. as if the vorticity were uniformly distributed over the surface between the two adjacent control points with density $\Gamma_j/2h$. For a flat plate this contribution is $\pi\Gamma_j/2h$. Values of the surface pressure for $N = 5$ are shown in figure 3(*b*).

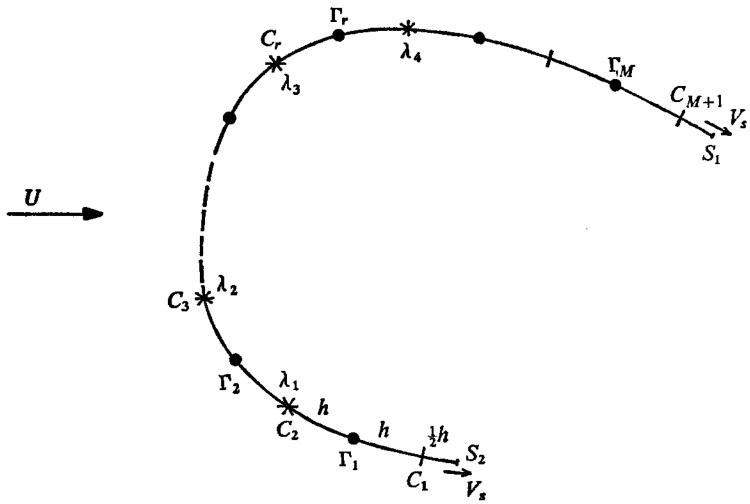

FIGURE 4. Vortex-lattice model for general two-dimensional body: M vortices designated Γ_r, 4 sources λ_r, $M+1$ control points C_r. Separation points at S_1, S_2. Equi-spaced distribution shown.

The above results were obtained by specifying equal spacing between vortex and control points. For thin aerofoils, Doe has extended the equi-spaced vortex analysis to general vortex spacing (Doe 1972), and a similar extension can be made for a normal flat plate. It can be shown that, in addition to the equi-spaced distribution, one with vortex points at

$$\left.\begin{array}{l} y = 1-\cos[\pi j/(N+1)], \quad j = 1, \ldots, N, \\ \text{and control points at} \\ y = 1-\cos[\pi(2i-1)/2(N+1)], \quad i = 1, \ldots, N+1, \end{array}\right\} \qquad (2.9)$$

will also give accurate results. This distribution has the advantage of allowing the sources to be placed closer to the separation points if this should prove necessary.

3. General two-dimensional and axisymmetric bodies

Owing to the form of the matrix of coefficients it is not possible to extend the analysis given in §2 to a general body shape. There is, however, no difficulty in applying the numerical method to other body shapes. On curved bodies the same distributions of vortex and control points are maintained on the wetted surface as on the flat plate but with the distance between points being measured along the surface (figure 4). Proof of the validity of the method must then come from a comparison with the analytic results of P & J, where this is possible, and from comparisons with measured pressure distributions.

It will be noted that in §2 it is the position of the source on the plate that is specified and not the base pressure. In the numerical procedure, the base pressure (i.e. the separation velocity $V_s = (1-c_{pb})^{\frac{1}{2}}$) is computed after the solution for the vortex and source strengths has been found. Any specified base pressure is obtained by iterating to the correct source position, since introducing source positions into the equations as unknowns would render them nonlinear. In our method, the sources must always be placed at control points, so that only the base pressures corresponding to these positions would be obtained. However this restriction can be overcome by replacing each of the sources by a pair of sources, placed at adjacent control points as in figure 4.

With M vortices there are $M+4$ unknowns and only $M+1$ control points at which to obtain equations. Two more equations can be obtained by specifying the tangential velocity at the separation points and another by assuming that the sum of the vortex strengths must be zero. (The latter is implicit in the assumption of symmetry for a normal flat plate.) We now have sufficient equations to find the unknowns and a solution satisfying the separation conditions can be obtained for any set of source positions. However, because the vorticity and hence the pressure distribution will be slightly different for each position of the sources, some method is needed for identifying the correct solution (i.e. the solution corresponding to the P & J solution, since this is known to give good agreement with experiment). By a consideration of the flow in the ζ plane of P & J, it can be shown that if the solution with source pairs is to satisfy the same separation conditions as the usual P & J solution with single sources then the two sources in a source pair will be positive if they straddle the single-source position. If they are not in the correct position then the source nearest to this position will be positive and the other negative. Using this result a simple iteration scheme was devised for finding the correct positions.

Evaluation of pressures

The pressure distribution on the wetted surface follows from knowledge of the surface velocity. As already noted, tangential velocities are evaluated at vortex points as the sum of contributions from the free stream, the sources and the other vortices and from treating the incumbent vortex as if it were distributed between the adjacent control points with uniform density $\gamma_j = \Gamma_j/s$, where s is the distance between the control points. For a general surface, this last contribution has been evaluated in the form

$$V_T = \pi - [\tfrac{1}{2}(\xi_1-\xi_2)\, d^2\eta/d\xi^2]_{\xi=0} + O((\xi_1-\xi_2)^3),$$

where (ξ, η) are local co-ordinates tangential and normal to the surface at the vortex point and the suffixes 1 and 2 refer to the adjacent control points.

The procedure for finding the normal velocity due to a source at its own control point is similar.

Asymmetric two-dimensional flow

Flow around asymmetric bluff bodies will in general give rise to a lift force, and a new condition needs to be found to specify the total circulation about the body. This condition could employ additional empirical information or involve a further assumption about the flow, although it is not immediately obvious what this should be. It should be noted that, because the free-streamline method does not model the flow within the separated region realistically, there is no simple relationship between the circulation about the body in the real flow and that about the vortex distribution representing the body. Thus the condition of zero total vorticity, already employed for symmetric bodies, does not necessarily imply zero circulation in the real flow. Arbitrarily fixing the value of the total vorticity in order to avoid the introduction of an additional empirical parameter is necessarily only an approximation for most lifting bodies but, as shown later, we have found that good predictions of the pressures on the wetted surface can be obtained for many lifting bodies with the zero-total-vorticity condition retained. This is not so surprising, perhaps, since the source strengths and positions are chosen so as to model the given separation conditions.

This approach can sometimes fail, however, because it becomes impossible to satisfy these conditions with positive sources located at the control points. That is, using the iteration scheme described earlier, one source pair is moved to the two control points next to a separation point. The source furthest away from the separation point is then still negative, indicating that the correct source position is closer to the separation point than the last control point. In many cases, the situation can be retrieved by using the cosine distribution given earlier or by increasing the number of vortices. However, in some cases, with high lift generated on the wetted surface, the number of vortices needed would be prohibitive. Further, we have found deteriorating agreement with experiment for bodies generating substantial lift. This suggests that the zero-total-vorticity condition is not appropriate for such cases. To obtain accurate results for these bodies an additional empirical parameter is required. Since we are not able to specify the net vorticity in the model *a priori*, we have chosen to use a pressure measured at some point on the wetted surface.

Axisymmetric flow

The extension of the method to axisymmetric flow is quite straightforward. The point singularities used in the two-dimensional method are replaced by ring vortices and ring sources and the appropriate induced velocities used. Otherwise the procedure is similar to that for symmetric two-dimensional bodies.

4. Results

Figure 5 presents results for a circular cylinder for two combinations of the base pressure and separation angle, using 29 vortices and the equi-spaced distribution. The pressures given by the numerical method are almost identical with those from the P & J theory and both agree well with the experimental results. (It may be noted that for bodies, such as the circular cylinder, experiencing separation from a continuous curved surface, the present method, in common with all free-streamline theories, will give rise to the physically unreal situation of an infinite favourable pressure gradient at separation for certain combinations of the base pressure and separation position (see Woods 1955). In general, such cases can only be distinguished *a posteriori*.)

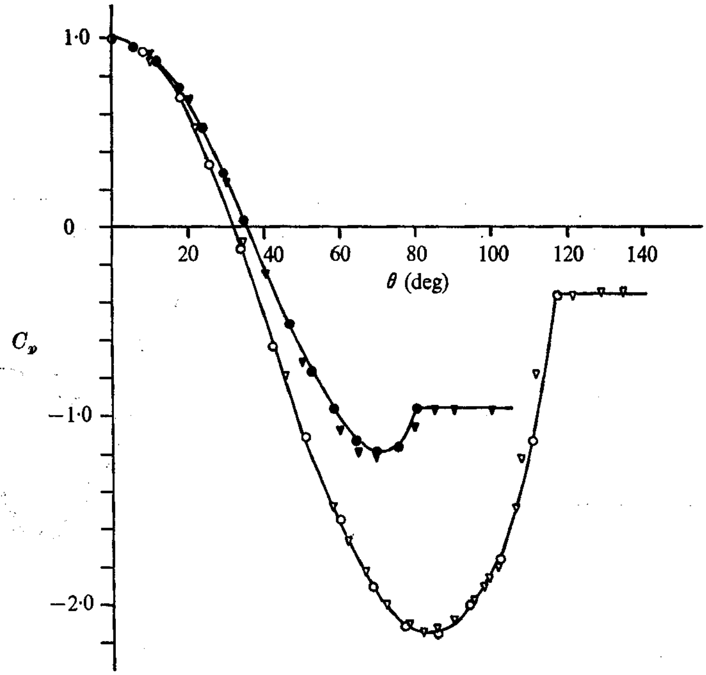

FIGURE 5. Circular cylinder. ——, P & J theory; ▼, experiment, Roshko (1954); ●, numerical, 29 vortices, equi-spaced, $c_{pb} = -0{\cdot}96$, separation at 80°; ▽, experiment, Bearman (1968); ○, numerical, 29 vortices, equi-spaced, $c_{pb} = -0{\cdot}38$, separation at 117·5°.

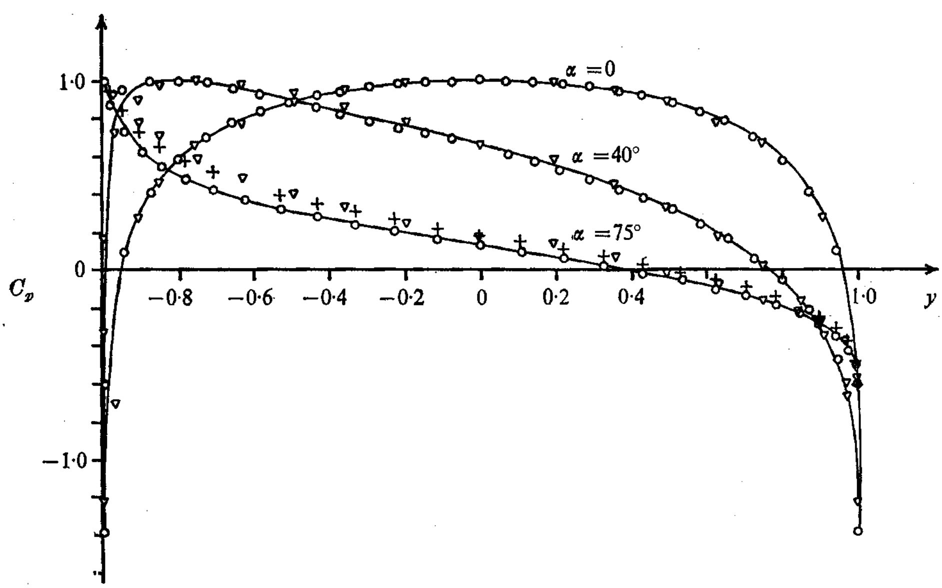

FIGURE 6. Flat plate. $\alpha = 0$, $c_{pb} = 1{\cdot}38$: ——, theory; ○, numerical equi-spaced distribution; ▽, experiment, Fage & Johansen (1927). $\alpha = 40°$, $c_{pb} = -1{\cdot}22$: ——, theory; ○, numerical, equi-spaced; ▽, experiment. $\alpha = 75°$, $c_{pb} = -0{\cdot}60$: ——, theory; ○, numerical, cosine distribution; +, numerical, cosine distribution with pressure at $y = 0$ specified; ▽, experiment. 27 vortices used.

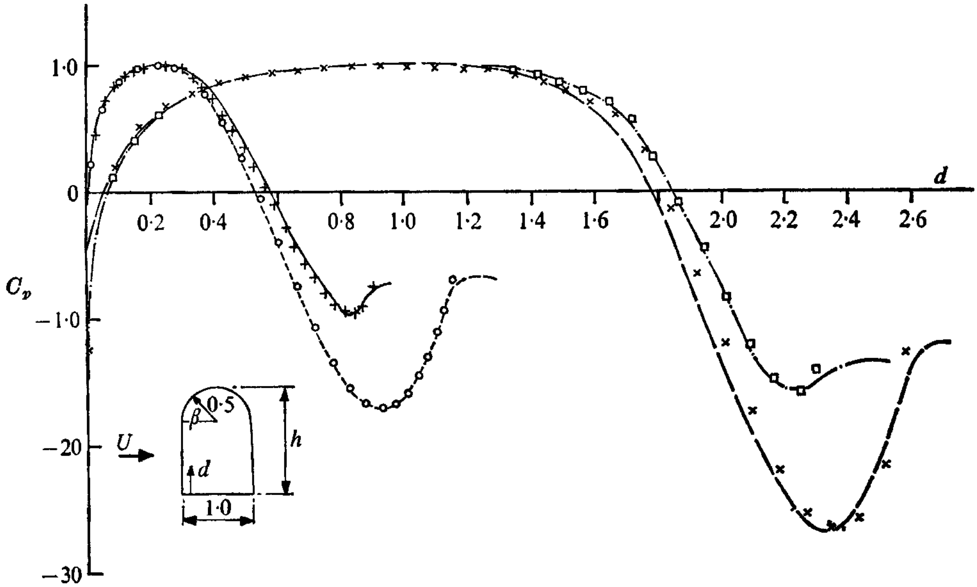

FIGURE 7. D-shaped bodies. $h = 2·25$, $\beta = 93°$, $c_{pb} = -1·25$: ×, numerical, equi-spaced; — —, experiment Bostock & Mair (1972). $h = 2·25$, $\beta = 61°$, $c_{pb} = -1·40$: □, numerical, equi-spaced; —·—, experiment. $h = 0·75$, $\beta = 74°$, $c_{pb} = -0·72$: +, numerical, equi-spaced; ——, experiment. $h = 0·75$, $\beta = 103°$, $c_{pb} = -0·68$: ○, numerical, cosine distribution with pressure at $d = 0·6$ specified; - - -, experiment. 30 vortices used. (Some points omitted for clarity.)

In figure 6 results are given for a flat plate for three incidence angles α. It is possible to extend P & J's theory for a normal flat plate to include the inclined case (Davies 1974) and a comparison is made with this theory and with the experimental values of Fage & Johansen (1927). The results for a normal flat plate are very good and with the condition of zero total vorticity retained in the numerical model (and implicitly in the theory) fairly good results can be obtained over the whole range of incidences. However there is deteriorating agreement between the experimental results and the theoretical and numerical results with increasing α and, at $\alpha = 75°$, the cosine distribution has to be used since the upstream source pair is very close to the separation point. For comparison, for this particular case results obtained by dropping the zero-vorticity condition and instead specifying the pressure at $y = 0$ are included. These show better agreement with experiment and give some indication of the approximation inherent in the use of the zero-vorticity condition.

Figure 7 gives numerical results for two D-shaped bodies with asymmetric separation, compared with the experimental results of Bostock & Mair (1972). The equi-spaced distribution has been employed and the zero-total-vorticity condition applied, except for the case with $h = 0·75$ and $c_{pb} = 103°$. In this case, the source pair nearest to the separation point on the curved surface had to be placed so close to the separation point that the inner member was still negative when 60 vortices in a cosine distribution were used. Hence the results shown were obtained by specifying the pressure at $d = 0·6$.

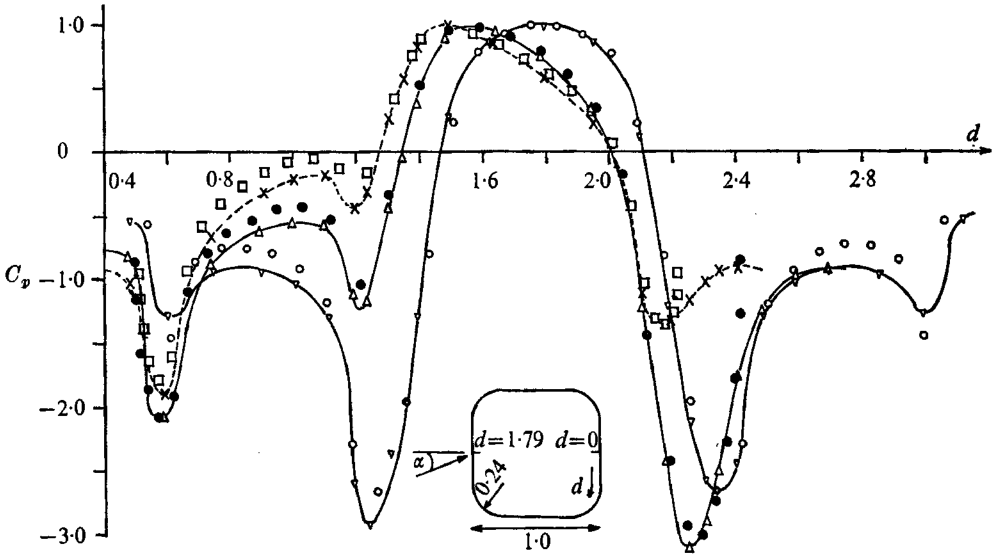

FIGURE 8. Square body with rounded corners. $\alpha = 0$, $Re = 4{\cdot}4\times10^5$, $c_{pb} = -0{\cdot}55$: —▽—, experiment, Polhamus *et al.* (1959); ○, numerical, equi-spaced, separation at $d = 0{\cdot}53$ and $d = 3{\cdot}06$. $\alpha = 20°$, $Re = 6{\cdot}2\times10^5$, $c_{pb} = -0{\cdot}85$: —△—, experiment; ●, numerical, cosine distribution, separation at $d = 0{\cdot}50$ and $d = 2{\cdot}41$. $\alpha = 30°$, $Re = 6{\cdot}2\times10^5$, $c_{pb} = 0{\cdot}95$: – – × – –, experiment; □, numerical, cosine distribution, separation at $d = 0{\cdot}50$ and $d = 2{\cdot}21$. 30 vortices used.

Somewhat more complicated pressure distributions can be found in the results of Polhamus, Geller & Grunwald (1959) for a square cylinder with rounded corners. These results cover a wide range of Reynolds number and incidences, yet the numerical method has given fairly good agreement with them. Some examples are given in figure 8.

Figure 9 shows results for axisymmetric flow past a circular disk. Agreement with the experimental values of Fail, Lawford & Eyre (1957) is very good using only 18 vortex rings. Figure 10 gives results for a sphere for two values of the Reynolds number. Again agreement with experiment (Fage 1937) is good. These last two sets of results illustrate that the use of sources to model separation conditions works as well for axisymmetric bluff-body flow as for two-dimensional flow.

An advantage of the vortex-lattice approach over other possible numerical techniques, such as the surface-source method of Hess & Smith (1967), is that, because only a few vortices are needed for accurate results, computing times are short. The results given required an average time of about 5 s on a CDC 6400 and a core size of 21K.

5. Conclusions

The method presented here is a generalization of Parkinson & Jandali's wake source model. It extends the application of their model to arbitrary two-dimensional and axisymmetric bluff body shapes by representing the wetted surface

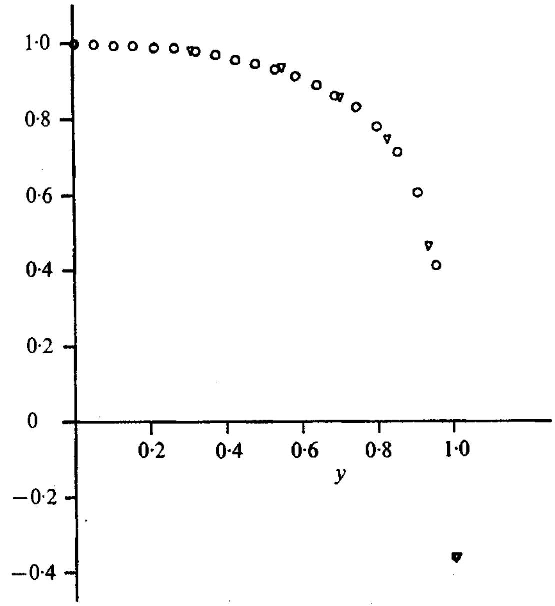

FIGURE 9. Circular disk. ▽, experiment, Fail *et al.* (1957); ○, numerical, 18 vortex rings, $c_{pb} = -0{\cdot}36$.

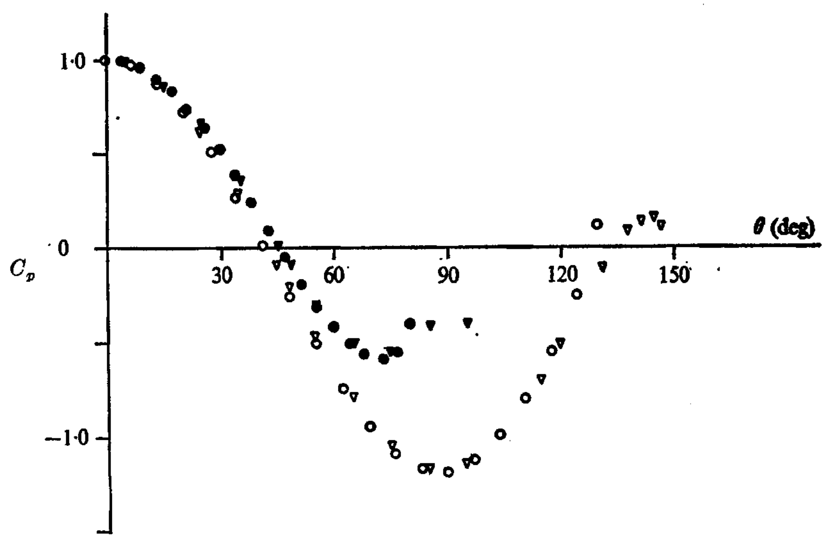

FIGURE 10. Sphere. ▼, experiment, Fage (1937); $Re = 1{\cdot}1 \times 10^5$; ●, numerical, 18 vortex rings, $c_{pb} = -0{\cdot}4$, separation at 80°; ▽, experiment, $Re = 4{\cdot}2 \times 10^5$; ○, numerical, 18 vortex rings, $c_{pb} = 0{\cdot}12$, separation at 130°.

by a distribution of discrete point vortices. In axisymmetric flow ring vortices and ring sources are used. The method is not restricted to a body in isolation and it would be possible, for example, to calculate the effects of the presence of the ground or of blockage in a wind tunnel by using a system of images.

Results obtained by the numerical technique agree closely with the analytic theory, for bodies to which the theory can be applied, and give good agreement with experiment for the other bodies examined, including several lifting bodies. There is deteriorating agreement as the lift increases, owing to the imposition of the zero-total-vorticity condition, and in some cases it is necessary to introduce an additional experimental parameter, the pressure at a point on the wetted surface, to allow this condition to be relaxed. The pressure distributions predicted for axisymmetric bluff bodies are in good agreement with experiment.

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